

6.2

Antidifferentiation by Substitution

DEFINITION Indefinite Integral

The family of all antiderivatives of a function $f(x)$ is the **indefinite integral of f with respect to x** and is denoted by $\int f(x)dx$.

If F is any function such that $F'(x) = f(x)$, then $\int f(x)dx = F(x) + C$, where C is an arbitrary constant, called the **constant of integration**.

EXAMPLE 1 Evaluating an Indefinite Integral

Evaluate $\int (x^2 - \sin x) dx$.

Properties of Indefinite Integrals

$$\int k f(x) dx = k \int f(x) dx \quad \text{for any constant } k$$

$$\int (f(x) \pm g(x)) dx = \int f(x) dx \pm \int g(x) dx$$

Power Formulas

$$\int u^n du = \frac{u^{n+1}}{n+1} + C \quad \text{when } n \neq -1$$

$$\int u^{-1} du = \int \frac{1}{u} du = \ln |u| + C$$

(see Example 2)

Trigonometric Formulas

$$\int \cos u du = \sin u + C$$

$$\int \sin u du = -\cos u + C$$

$$\int \sec^2 u du = \tan u + C$$

$$\int \csc^2 u du = -\cot u + C$$

$$\int \sec u \tan u du = \sec u + C$$

$$\int \csc u \cot u du = -\csc u + C$$

Exponential and Logarithmic Formulas

$$\int e^u du = e^u + C$$

$$\int a^u du = \frac{a^u}{\ln a} + C$$

$$\int \ln u du = u \ln u - u + C \quad (\text{See Example 2})$$

$$\int \log_a u du = \int \frac{\ln u}{\ln a} du = \frac{u \ln u - u}{\ln a} + C$$

EXAMPLE 2 Verifying Antiderivative Formulas

Verify the antiderivative formulas:

$$(a) \int u^{-1} du = \int \frac{1}{u} du = \ln |u| + C$$

$$(b) \int \ln u du = u \ln u - u + C$$

EXPLORATION 1 Are $\int f(u) du$ and $\int f(u) dx$ the Same Thing?

Let $u = x^2$ and let $f(u) = u^3$.

1. Find $\int f(u) du$ as a function of u .
2. Use your answer to question 1 to write $\int f(u) du$ as a function of x .
3. Show that $f(u) = x^6$ and find $\int f(u) dx$ as a function of x .
4. Are the answers to questions 2 and 3 the same?

EXAMPLE 3 Paying Attention to the Differential

Let $f(x) = x^3 + 1$ and let $u = x^2$. Find each of the following antiderivatives in terms of x :

(a) $\int f(x) dx$ (b) $\int f(u) du$ (c) $\int f(u) dx$

U Substitution Activity

EXAMPLE 4 Using Substitution

Evaluate $\int \sin x e^{\cos x} dx$.

EXAMPLE 5 Using Substitution

Evaluate $\int x^2 \sqrt{5 + 2x^3} dx$.

EXAMPLE 6 Using Substitution

Evaluate $\int \cot 7x \, dx$.

EXAMPLE 7 Setting Up a Substitution with a Trigonometric Identity

Find the indefinite integrals. In each case you can use a trigonometric identity to set up a substitution.

(a) $\int \frac{dx}{\cos^2 2x}$ (b) $\int \cot^2 3x \, dx$ (c) $\int \cos^3 x \, dx$

EXAMPLE 8 Evaluating a Definite Integral by Substitution

Evaluate $\int_0^{\pi/3} \tan x \sec^2 x \, dx$.

EXAMPLE 9 That Absolute Value Again

Evaluate $\int_0^1 \frac{x}{x^2 - 4} dx$.